ISTEK BILGE KAGAN ANATOLIAN HIGH SCHOOL
Diploma Programme / MATHEMATICS

A Research Proposal on

Analysis of Neural Complexity in ADHD: A Fractal Dimension Study of Resting-State EEG Signals

Research Question : To what extend fractal dimensions of multichannel (19 channel) resting-state EEG time series different between the groups with an ADHD diagnosis and neurotypical controls, and can these measures pose as a biomarker for neural-complexity changes in ADHD

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Contents

1 Introduction **1**

1.1 Research Question 1

1.2 Research Context 1

1.3 Problem Statement 2

2 Theory of Fractal Dimension Methods **3**

2.0.1 The Mathematics of Higuchi's Fractal Dimension (HFD) 3

2.0.2 Standard Deviation 4

2.0.3 Machine Learning for Classification 5

2.0.4 Random Forest Algorithm 5

2.0.5 Justification for Choosing HFD and RF Algorithms 9

3 Preliminary Literature Review by ELICIT **10**

1. Introduction

An electroencephalogram (EEG) is a test that measures electrical activity in the brain. This test also is called an EEG. The test uses small, metal discs called electrodes that attach to the scalp. Brain cells communicate via electrical impulses, and this activity shows up as wavy lines on an EEG recording. Brain cells are active all the time, even during sleep. An EEG is one of the main tests to help diagnose epilepsy. An EEG also can play a role in diagnosing other brain conditions. (?, ?) Furthermore, ADHD (Attention Deficit and Hyperactivity Disorder) is one of the most encountered and misdiagnosed neurodevelomental disorders due to subjective diagnosis. Furthermore, misdiagnosis of ADHD and overprescribed of ADHD drugs can cause serious abuseses.

1.1 Research Question

In this study, the research question is: 'To what extent fractal dimensions of multichannel (19-channel) resting-state EEG time series differ between the groups with an ADHD diagnosis and neurotypical controls, and can these measures serve as a biomarker for neural-complexity changes in ADHD?' This study will contain experiments on datasets of ADHD and neurotypical controls' EEG data to observe differentiation and diagnosis of ADHD.

1.2 Research Context

This study aims to investigate whether fractal dimension calculation methods are effective in EEG signal analysis for detecting ADHD. In this study, applied mathematics is applied to the field of neurology. Furthermore, Katz and Higuchi's fractal dimension (FD) analysis will be the primary method used to analyze the EEG signals of control and ADHD individuals. Also, machine learning methods like Random Forest can be used to classify the ADHD and controls with the features of HFD (Higuchi Fractal Dimension) and KFD (Katz Fractal Dimension)

1.3 Problem Statement

The hardest thing in this and can be perceived as a problem is finding an ADHD individuals dataset, but I have one to do experiments

2. Theory of Fractal Dimension Methods

2.0.1 The Mathematics of Higuchi's Fractal Dimension (HFD)

The Higuchi method is a technique used to calculate the fractal dimension of a time series and is effective in measuring the complexity of EEG signals.

Sub-Series Construction

For a time series $x(1), x(2), ..., x(N)$ of length N, sub-series are constructed as follows:

1. **Step 1:** Determine the $k_{\max}$ value (chosen as 10 in this study).
2. **Step 2:** For each k value (from $k = 1, 2, ..., k_{\max}$):
    - Select starting points $m = 1, 2, ..., k$.
    - For each m, create a sub-series with a step interval of k:

$$x_k^m = \left\{ x(m), x(m+k), x(m+2k), ..., x\left(m + \left\lfloor \frac{N-m}{k} \right\rfloor k\right) \right\}$$

Length Calculation Formula

For each sub-series, the length is calculated as:

$$L_m(k) = \frac{1}{k} \left[\left(\sum_{i=1}^{\lfloor \frac{N-m}{k} \rfloor} |x(m+ik) - x(m+(i-1)k)| \right) \frac{N-1}{\lfloor \frac{N-m}{k} \rfloor k} \right]$$

The total length for a given k, denoted $L(k)$, is the average of the lengths $L_m(k)$ over all starting points m. The HFD is the slope of the line that best fits the points $(\ln(1/k), \ln(L(k)))$ via linear regression.

Example Calculation

For a 12-point EEG signal ($N = 12$) with $k_{\max} = 4$: $X = \{0.1, 0.4, 0.2, 0.5, 0.3, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1$

For k=2:

- Sub-series for $m = 1$: $\{0.1, 0.2, 0.3, 0.7, 0.9, 1.1\}$

- Sub-series for $m = 2$: $\{0.4, 0.5, 0.6, 0.8, 1.0, 1.2\}$
- $L_1(2) = 0.44$, $L_2(2) = 0.33$
- Average Length $L(2) = (0.44 + 0.33)/2 = 0.385$

Regression Analysis for HFD Calculation

The values are plotted on a log-log scale to find the slope (the HFD).

Table 2.1: Values and intermediate products used for regression.

k	$L(k)$	$\ln(1/k)$ (x)	$\ln(L(k))$ (y)	$x \cdot y$
1	1.6000	0.0000	0.4700	0.0000
2	0.3850	-0.6931	-0.9544	0.6616
3	0.4006	-1.0986	-0.9149	1.0049
4	0.1977	-1.3863	-1.6213	2.2477

Calculation of Totals:

$$\sum x = -3.1780$$
$$\sum y = -3.0206$$
$$\sum xy = 3.9142$$
$$\sum x^2 = 3.6091$$

Slope (HFD) (b) Calculation:

$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{4 \times 3.9142 - (-3.1780) \times (-3.0206)}{4 \times 3.6091 - (-3.1780)^2} = 1.397$$

The resulting HFD is 1.397.

2.0.2 Standard Deviation

Standard deviation is the square root of the average of the squared distances of individual values from the arithmetic mean. In EEG signals, it is used to indicate the overall amount of fluctuation and power of the signal. A low standard deviation indicates that the signal is less variable, while a high standard deviation indicates that the signal is more fluctuating and

variable. The formula for standard deviation ('std') is:

$$std = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

In this study, the standard deviations of the EEG signals were analyzed along with the Higuchi fractal dimension to evaluate both the complexity level and the variance of the signal together.

2.0.3 Machine Learning for Classification

SMOTE (Synthetic Minority Over-sampling Technique)

SMOTE is a technique used to balance the dataset. Since there was an imbalance in the dataset of patients with ADHD, synthetic samples were created using SMOTE. The algorithm operates as follows:

1. For each instance in the minority class, find its k nearest neighbors (in our system, $k = 1$).
2. Randomly select one of these neighbors.
3. Create a new synthetic instance between the original instance and the selected neighbor:

   $$x_{\text{new}} = x_i + \lambda \times (x_{zi} - x_i)$$

   Here, λ is a random number in the range [0,1].

As a result of applying SMOTE, the class distribution in the training dataset was balanced, and the model's ability to learn the minority class was enhanced.

2.0.4 Random Forest Algorithm

The following table explains the basic parameters used in the matrix-based decision tree generation algorithm and their roles in the algorithm.

Parameter	Symbol	Description (Theory)	In Our System
Input Matrix	$\mathbf{X}_t$	Feature matrix obtained through bootstrap sampling.	`X` or `X_train_resampled` (EEG features)
Target Vector	$\mathbf{y}_t$	Target values corresponding to observations.	`y` or `y_train_resampled` (ADHD:1, healthy:0)
Feature Subset Size	m	Number of features to select for splitting at a node.	`max_features` (default: $\sqrt{p}$)
Feature Amount	p	Total number of features in the original data.	`X.shape[1]` (number of EEG features)
Current Sample Set	$\mathbf{S}$	Subset of samples reaching the node.	Current group of individuals in the tree
Selected Features	$\mathbf{F}$	Randomly selected features for splitting.	Randomly selected m EEG features at each split
Feature Value	f	Single feature selected for splitting.	One of the `X` columns (e.g., `theta_beta_ratio`)
Threshold Value	v	Value used to split the data into two.	Split point in EEG feature (e.g., 2.5)
Left Subset	$\mathbf{S}_L$	Samples less than or equal to the threshold.	Individuals going to the left branch
Right Subset	$\mathbf{S}_R$	Samples greater than the threshold.	Individuals going to the right branch
Trained Tree	h_t	Tree trained on the bootstrap sample.	Individual trees in `RandomForestClassifier`

In the Random Forest algorithm, each tree is created by performing operations on the data matrix and feature matrix. The process steps are as follows:

1. **Training Data Matrix**:

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

Here n is the number of observations and p is the number of features.

2. **Bootstrap Sampling Matrix**:

$$\mathbf{X}_t = \begin{pmatrix} x_{11}^* & x_{12}^* & \cdots & x_{1p}^* \\ x_{21}^* & x_{22}^* & \cdots & x_{2p}^* \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}^* & x_{n2}^* & \cdots & x_{np}^* \end{pmatrix}, \quad \mathbf{y}_t = \begin{pmatrix} y_1^* \\ y_2^* \\ \vdots \\ y_n^* \end{pmatrix}$$

The starred elements are selected from the original matrix through resampling with replacement.

3. **Random Feature Selection**: For each node split, $m \approx \sqrt{p}$ features are randomly selected:

$$\mathbf{F}_{\text{subset}} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{pmatrix}, \quad m < p$$

4. **Split Criterion Calculation** (for Gini index):

$$\text{Gini}(S) = 1 - \sum_{k=1}^{K} (p_k)^2$$

Here p_k is the ratio of the k-th class in split S.

5. **Split Point Optimization**:

$$\text{Gain} = \text{Gini}(S) - \left(\frac{|S_L|}{|S|} \text{Gini}(S_L) + \frac{|S_R|}{|S|} \text{Gini}(S_R) \right)$$

S_L and S_R are the samples in the left and right child nodes.

[1] **Input:** $\mathbf{X}_t$ bootstrap matrix, $\mathbf{y}_t$ target vector, m feature subset size Create root node with all data ($S = \mathbf{X}_t$) stopping criterion not met Get samples in current node: $\mathbf{S} \subseteq \mathbf{X}_t$ Randomly select m features: $\mathbf{F} \subset \{1, \ldots, p\}$ each feature $f \in \mathbf{F}$ each threshold value v (based on feature values) Split data: $\mathbf{S}_L = \{\mathbf{x} \in \mathbf{S} | x_f \leq v\}$, $\mathbf{S}_R = \{\mathbf{x} \in \mathbf{S} | x_f > v\}$ Calculate Gini gain Select best (f, v) pair Split node into two child nodes with $\mathbf{S}_L$ and $\mathbf{S}_R$ **Output:** Trained decision tree h_t

Mathematical Example Let's consider a simple example with 2 features:

$$\mathbf{X} = \begin{pmatrix} 1 & 5 \\ 2 & 4 \\ 3 & 3 \\ 4 & 2 \\ 5 & 1 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Step 1: For the first split, randomly select 1 of the 2 features ($m = 1$). Suppose f_1 is selected.

Step 2: Possible threshold values: 1.5, 2.5, 3.5, 4.5

Step 3: For $v = 2.5$:

$$\begin{aligned} \mathbf{S}_L &= \{\mathbf{x} | x_1 \leq 2.5\} \Rightarrow \text{first two samples} \\ \mathbf{S}_R &= \{\mathbf{x} | x_1 > 2.5\} \Rightarrow \text{last three samples} \\ \text{Gini}(S) &= 1 - \left(\frac{2}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = 0.48 \\ \text{Gini}(S_L) &= 0 \quad (\text{all classes } 0) \\ \text{Gini}(S_R) &= 1 - \left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2 \approx 0.444 \\ \text{Gain} &= 0.48 - \left(\frac{2}{5} \times 0 + \frac{3}{5} \times 0.444\right) \approx 0.2136 \end{aligned}$$

This process is repeated for all possible splits, and the split with maximum gain is selected.

2.0.5 Justification for Choosing HFD and RF Algorithms

The main scientific justifications for selecting Higuchi Fractal Dimension (HFD) and Random Forest (RF) algorithms in this study are as follows:

Reasons for Choosing Higuchi Fractal Dimension (HFD)

- **Suitability for Nonlinear Signal Analysis:** Due to the inherently non-stationary and nonlinear nature of EEG signals, traditional linear methods (FFT, wavelet) are inadequate (?, ?). HFD:

$$HFD = \frac{\log(L(k))}{\log(1/k)} \tag{2.1}$$

  can measure the complexity level of the signal based on fractal geometry.

- **Neurodynamic Structure of ADHD:** It is known that the neural network topology in the frontal lobe and basal ganglia of individuals with ADHD exhibits fractal properties (?, ?). The primary reason for choosing HFD is its capacity to quantitatively measure this pathological fractal structure.

- **Computational Efficiency:** HFD's $\mathcal{O}(N)$ computational complexity makes it suitable for real-time clinical applications, and it provides more stable results compared to other fractal methods (Katz, Petrosian).

3. Preliminary Literature Review by ELICIT

Using research question ”To what extend fractal dimensions of multichannel (19 channel) resting-state EEG time series different between the groups with an ADHD diagnosis and neurotypical controls, and can these measures pose as a biomarker for neural-complexity changes in ADHD”, Elicit searched across over 126 million academic papers from the Semantic Scholar corpus. Elicit retrieved the 499 papers most relevant to the query. Report going to be shared with the proposal.

To what extend fractal dimensions of multichannel (19 channel) resting-state EEG time series different between the groups with an ADHD diagnosis and neurotypical controls, and can these measures pose as a biomarker for neural-complexity changes in ADHD

Fractal and complexity measures from 19-channel resting-state EEG recordings significantly distinguish ADHD from neurotypical controls with classification accuracies of 83-100%, indicating these measures can serve as biomarkers for ADHD-related neural complexity changes.

Abstract

Fractal and complexity measures derived from resting-state EEG recordings distinguish ADHD from neurotypical controls. In several studies using 19-channel recordings, approximate entropy and multifractal metrics recorded from frontal and prefrontal regions were lower in ADHD, whereas one study found higher Lempel–Ziv complexity at temporal electrodes ($p < 0.01$). Other measures—including multifractal detrended fluctuation analysis, largest Lyapunov exponent, q-statistics, and fuzzy entropy—differentiated groups with statistical significance. In addition, classifiers based on these features achieved prediction accuracies ranging from approximately 83% up to nearly 100% under cross-validation.

Notable observations include:

1. Frontal and prefrontal areas yielded the most consistent group differences.
2. Both decreases and isolated increases in complexity were reported, depending on the metric and region.
3. High classification performance in several studies supports the view that these nonlinear measures reflect neural-complexity changes associated with ADHD.

These findings indicate that fractal and complexity analyses of multichannel 19-channel EEG recordings capture aspects of neural dynamics that may serve as biomarkers of ADHD-related alterations in brain function.

Paper search

Using your research question "To what extend fractal dimensions of multichannel (19 channel) resting-state EEG time series different between the groups with an ADHD diagnosis and neurotypical controls, and can these measures pose as a biomarker for neural-complexity changes in ADHD", we searched across over 126 million academic papers from the Semantic Scholar corpus. We retrieved the 499 papers most relevant to the query.

Screening

We screened in sources that met these criteria:

- **ADHD vs Control Comparison**: Does this study compare individuals with ADHD diagnosis to neurotypical controls?
- **Fractal Dimension Measures**: Does this study measure fractal dimensions or fractal-based complexity measures from EEG time series?
- **Multichannel Resting-State EEG**: Does this study use multichannel EEG with at least 19 channels during resting-state conditions?
- **Clinical ADHD Diagnosis**: Does this study include participants with clinically diagnosed ADHD?

- **Quantitative Reporting**: Does this study report quantitative fractal dimension values or complexity metrics?
- **Appropriate Study Design**: Is this study a cross-sectional, case-control, cohort study, systematic review, or meta-analysis?
- **Sufficient Publication Quality**: Is this study NOT a case report, editorial, conference abstract, or opinion piece?
- **Human Research**: Is this study conducted on human participants (not animal or in vitro research)?

We considered all screening questions together and made a holistic judgement about whether to screen in each paper.

Data extraction

We asked a large language model to extract each data column below from each paper. We gave the model the extraction instructions shown below for each column.

- **Study Design**:

  Describe the overall study design. Specifically:

- Type of study (e.g., cross-sectional observational)
- Whether it was a comparative study between ADHD and control groups
- Specify the data collection method (e.g., resting-state EEG during eyes-closed condition)
- Indicate if this was a single-center or multi-center study

If information is unclear or partially missing, note "Insufficient information" and provide any available details.

- **Participant Characteristics**:

  Extract the following participant details:

- Total sample size
- Number of ADHD participants
- Number of control participants
- Age range or mean age (with standard deviation)
- Age matching between groups
- Gender distribution
- Diagnostic criteria used for ADHD (if mentioned)

If any information is missing, use "NR" (Not Reported). If ranges or means are provided, record both.

- **EEG Recording Parameters**:

  Record specific details about EEG data collection:

- Number of EEG channels (confirm 19-channel)
- Recording conditions (e.g., eyes-closed resting state)
- EEG recording duration
- Equipment/system used (if specified)
- Sampling rate (if reported)

If any parameter is not clearly stated, mark as "NR" (Not Reported).

- **Fractal and Complexity Analysis Methods**:

  Systematically extract:

- Specific fractal/complexity measures used (e.g., multifractal singularity spectrum, Lempel-Ziv Complexity)
- Frequency bands analyzed
- Statistical methods for comparing group differences
- Classification methods and their performance (accuracy, sensitivity, specificity)

Provide exact numerical values when available. If multiple methods were used, list all.

- **Primary Outcome Measures**:

  Extract:

- Statistically significant differences in fractal/complexity measures between ADHD and control groups
- Specific brain regions showing significant differences
- P-values and effect sizes (if reported)
- Key conclusions about neural complexity differences

If no statistically significant results were found, clearly state "No significant differences detected".

Results

Characteristics of Included Studies

Study	Study Design	Sample Size (ADHD/Controls)	EEG Channels	Fractal/Complexity Measures Used	Full text retrieved
Khoshnoud et al., 2017	Cross-sectional observational	12/12	19	Multifractal singularity spectrum, largest Lyapunov exponent, approximate entropy	No
Khoshnoud et al., 2018	Cross-sectional observational	12/12	19	Multifractal singularity spectrum, largest Lyapunov exponent, approximate entropy	No
Asayesh et al., 2024	Cross-sectional observational	30/30	19	Lempel-Ziv Complexity (LZC)	No

Study	Study Design	Sample Size (ADHD/Controls)	EEG Channels	Fractal/Complexity Measures Used	Full text retrieved
Khoshnoud et al., "Functional analysis of ADHD in children"	Cross-sectional observational	12/12	19	Multifractal singularity spectrum, largest Lyapunov exponent, approximate entropy	No
Khoshnoud et al., 2020	Cross-sectional observational	No mention found	No mention found	Multifractal detrended fluctuation analysis (MF-DFA)	No
Roozbehi et al., 2020	Cross-sectional observational	16/16	No mention found	q-order Hurst exponent, classical scaling exponent, singularity spectrum (multifractal detrended fluctuation analysis)	No
Fernandez-Quintana et al., 2019	Cross-sectional observational	50/58	No mention found	Complexity (unspecified)	No
Abramov et al., 2024	Cross-sectional observational	19/19	20	q-statistics (stretched q-exponential, exponent c)	Yes
Chen et al., 2019	Cross-sectional observational	50/58	No mention found	Complexity (unspecified)	No
Khoshnoud et al., 2015	Cross-sectional observational	No mention found	No mention found	Largest Lyapunov Exponent, Approximate Entropy	No
Arnett et al., 2022a	Cross-sectional observational	88/29	No mention found	Aperiodic spectral slope	No
Sebastián et al., 2004	Cross-sectional observational	No mention found	No mention found	Fractal interpolation dimension	No
Garc'ia-Ponsoda et al., 2024	Observational (secondary data)	No mention found	No mention found	Katz fractal dimension	No

Study	Study Design	Sample Size (ADHD/Controls)	EEG Channels	Fractal/Complexity Measures Used	Full text retrieved
Arunkumar et al., 2024	Cross-sectional observational	No mention found	16	Nonlinear fractal dimensions approach	No
Arnett et al., 2022b	Cross-sectional observational	29/30	128	Aperiodic spectral slope (exponent)	Yes
Angulo-Ruiz et al., 2022	Cross-sectional observational	32/No mention found (open eyes), 25/No mention found (closed eyes)	32	Multiscale entropy, standard deviation, coefficient of variation of power spectral density	Yes
Gu et al., 2022	Cross-sectional observational	30/30	No mention found	Multiscale entropy	No
Valdizán et al., 1997	Cross-sectional observational	19/9	No mention found	Dimension of correlation (D2)	No
Chen et al., 2024	Cross-sectional observational	43/35	No mention found	Entropy estimators (unspecified)	No
Mao et al., 2025	Cross-sectional observational	61/60	19	Fuzzy entropy	Yes
Ginard Puigserver, 2016	Cross-sectional observational	9/9	No mention found	Multiscale entropy, sample entropy, fractal dimension	No
Arnett et al., 2021	Cross-sectional observational	88/29	No mention found	Aperiodic spectral slope	No
Pham et al., 2021	Cross-sectional observational	No mention found	No mention found	Fractal dimensions	No
Arnett et al., 2024	Cross-sequential cohort	No mention found	No mention found	Aperiodic exponent	No

Study	Study Design	Sample Size (ADHD/Controls)	EEG Channels	Fractal/Complexity Measures Used	Full text retrieved
Cura et al., 2022	Cross-sectional observational	15/18	30	Largest Lyapunov Exponent, correlation dimension, Hurst exponent, Katz fractal dimension, Higuchi fractal dimension, approximate entropy	Yes

Fractal and complexity measures used in EEG studies of ADHD:

- Entropy-based measures (approximate entropy, multiscale entropy, sample entropy, fuzzy entropy, approximate entropy): 9 studies
- Multifractal analysis (including multifractal detrended fluctuation analysis, singularity spectrum): 5 studies
- Lyapunov exponent : 5 studies
- Fractal dimension-based measures (including Katz, Higuchi, correlation dimension, D2, nonlinear fractal dimension): 7 studies
- Aperiodic exponent or spectral slope : 4 studies
- Lempel-Ziv Complexity : 1 study
- Hurst exponent : 1 study
- q-statistics : 1 study
- Unspecified complexity : 2 studies
- Unspecified entropy : 1 study

EEG channel counts:

- 19 channels: 5 studies
- 20 channels: 1 study
- 16 channels: 1 study
- 30 channels: 1 study
- 32 channels: 1 study
- 128 channels: 1 study
- For 15 studies, we didn't find mention of EEG channel information

Sample size reporting:

- We found mention of sample size for 18 studies, with ADHD group sizes ranging from 9 to 88 and control group sizes from 9 to 60
- For 7 studies, we didn't find mention of sample size

Effects

Fractal Dimension Differences Between Groups

Study	Complexity Measure	Brain Region/Channels	Effect Direction (ADHD vs. Control)	Statistical Significance
Khoshnoud et al., 2017	Largest Lyapunov exponent, approximate entropy, multifractal spectrum	Left frontal-central, prefrontal	Lower approximate entropy in ADHD; altered spectrum	Significant (Wilcoxon, p-value not reported)
Khoshnoud et al., 2018	Same as above	Same as above	Same as above	Significant (Wilcoxon, p-value not reported)
Asayesh et al., 2024	Lempel-Ziv Complexity	Temporal electrodes	Higher complexity in ADHD	$p < 0.01$
Khoshnoud et al., "Functional analysis..."	Same as Khoshnoud 2017	Left frontal-central, prefrontal	Lower approximate entropy in ADHD; altered spectrum	Significant (Wilcoxon, p-value not reported)
Khoshnoud et al., 2020	Multifractal detrended fluctuation analysis	Prefrontal, mid-frontal, right frontal (source)	Decreased multifractality in ADHD	Significant (p-value not reported)
Roozbehi et al., 2020	Multifractal detrended fluctuation analysis features	Channels 16, 2	Individual differences; discriminative	Implied by high accuracy
Fernandez-Quintana et al., 2019	Complexity	No mention found	Lower complexity in ADHD	Significant (p-value not reported)
Abramov et al., 2024	q-statistics (c, q)	Global	Higher complexity in ADHD	$p < 0.01$
Chen et al., 2019	Complexity	No mention found	Lower complexity in ADHD	Significant (p-value not reported)
Khoshnoud et al., 2015	Largest Lyapunov Exponent, approximate entropy	Temporo-frontal	Lower approximate entropy in ADHD	Significant (ANOVA, p-value not reported)
Arnett et al., 2022a	Aperiodic spectral slope	Global	No modulation in ADHD; controls show flattening	Significant (p-value not reported)
Sebastián et al., 2004	Fractal interpolation dimension	No mention found	Lower in ADHD	Significant (p-value not reported)
Garc'ia-Ponsoda et al., 2024	Katz fractal dimension	P3, P4, C3	No mention found	No significant differences detected
Arunkumar et al., 2024	Nonlinear fractal dimensions	No mention found	No mention found	No significant differences detected

Study	Complexity Measure	Brain Region/Channels	Effect Direction (ADHD vs. Control)	Statistical Significance
Arnett et al., 2022b	Aperiodic spectral slope, P3a amplitude	Global	Non-responders: flatter slope; responders: attenuated P3a	Significant (area under the curve 0.79)
Angulo-Ruiz et al., 2022	Multiscale entropy, standard deviation, coefficient of variation	No mention found	Lower complexity, higher variability in ADHD	Significant (p-value not reported)
Gu et al., 2022	Multiscale entropy	Frontal	Lower entropy change in ADHD	Significant (p-value not reported)
Valdizán et al., 1997	Dimension of correlation (D2)	Occipital (task)	More activated areas in ADHD	Significant (p-value not reported)
Chen et al., 2024	Entropy estimators	Frontal	Lower entropy in ADHD-S	Significant (p-value not reported)
Mao et al., 2025	Fuzzy entropy, power spectral density, mutual information	Frontal, central, posterior	Increased theta/alpha/beta power, theta/beta ratio, connectivity in ADHD	Significant (p-value not reported)
Ginard Puigserver, 2016	Multiscale entropy, sample entropy, fractal dimension	Fz, Cz	Lower in ADHD (Fz > Cz)	Significant (p-value not reported)
Arnett et al., 2021	Aperiodic spectral slope	Global	Reduced slope in ADHD (lights off)	Significant (p-value not reported)
Pham et al., 2021	Fractal dimensions	No mention found	No mention found	No mention found
Arnett et al., 2024	Aperiodic exponent	Global	Decreased in ADHD	Significant (p-value not reported)
Cura et al., 2022	Largest Lyapunov Exponent, correlation dimension, Hurst exponent, Katz fractal dimension, Higuchi fractal dimension, approximate entropy	No mention found	Nonlinear features: higher accuracy	Significant (accuracy ≥90.3%)

Complexity measures used:

- Fractal or multifractal measures : 12 studies
- Entropy-based measures (including approximate entropy, multiscale entropy, sample entropy, fuzzy entropy, entropy estimators): 10 studies

- Other nonlinear measures (including Lyapunov exponent, mutual information, standard deviation, coefficient of variation, P3a amplitude): 8 studies
- Aperiodic spectral slope or exponent : 4 studies
- Lempel-Ziv Complexity and q-statistics : 1 study each
- ”Complexity” reported without further specification : 2 studies

Effect direction:

- Lower complexity, entropy, or fractality in ADHD compared to controls: 12 studies
- Higher complexity in ADHD: 2 studies
- Decreased multifractality in ADHD: 1 study
- No significant difference: 2 studies
- Other, mixed, or not clearly specified effects (e.g., altered spectrum, individual differences, or no mention found): 10 studies

Statistical significance:

- 21 studies reported a statistically significant difference between ADHD and controls for at least one complexity measure (as reported by the studies; p-values were not always provided)
- 2 studies reported no significant difference
- 1 study implied significance by high classification accuracy
- For 1 study, we didn't find mention of statistical significance

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Classification Performance and Biomarker Potential

Study	Feature Type	Classification Method	Accuracy	Cross-validation Results
Khoshnoud et al., 2017	Nonlinear features	Support Vector Machine, Radial Basis Function Neural Network	83.33%	4-fold cross-validation
Khoshnoud et al., 2018	Nonlinear features	Support Vector Machine, Radial Basis Function Neural Network	83.33%	4-fold cross-validation
Asayesh et al., 2024	Lempel-Ziv Complexity, power features	Support Vector Machine	78.02% ± 1.80%	No mention found
Khoshnoud et al., ”Functional analysis…”	Nonlinear features	Support Vector Machine, Radial Basis Function Neural Network	83.33%	4-fold cross-validation
Khoshnoud et al., 2020	Source-based multifractal detrended fluctuation analysis	Support Vector Machine	86.67%	No mention found

Study	Feature Type	Classification Method	Accuracy	Cross-validation Results
Roozbehi et al., 2020	Multifractal detrended fluctuation analysis features	Linear Support Vector Machine	94%, 97%	No mention found
Fernandez-Quintana et al., 2019	Complexity, spectral, bicoherence	Support Vector Machine	84.59%	Area under the curve 0.9158
Abramov et al., 2024	q-statistics (c, q)	Clustering	100% (task)	No mention found
Chen et al., 2019	Complexity, spectral, bicoherence	Support Vector Machine	84.59%	Area under the curve 0.9158
Khoshnoud et al., 2015	Largest Lyapunov Exponent, approximate entropy	Probabilistic Neural Network	87.5%	No mention found
Garc'ia-Ponsoda et al., 2024	Kurtosis, Katz fractal dimension, power spectral density	XGBoost, Support Vector Machine, k-Nearest Neighbors	No mention found	No mention found
Arunkumar et al., 2024	Nonlinear fractal dimensions	Convolutional Neural Network (ResNet34)	99.5%	10-fold cross-validation
Arnett et al., 2022b	Aperiodic slope, P3a	Receiver Operating Characteristic	Area under the curve 0.79	Sensitivity 0.71, Specificity 0.70
Mao et al., 2025	Fuzzy entropy, power spectral density, mutual information	CatBoost, others	99.58% (CatBoost)	10-fold cross-validation
Cura et al., 2022	Nonlinear features	Decision Tree, Naive Bayes, Support Vector Machine, k-Nearest Neighbors, Boosted Trees	≥90.3% (nonlinear), ≥77.9% (time)	10-fold cross-validation
Pham et al., 2021	Fractal dimensions	Ensemble learning	Higher than state-of-the-art	No mention found

Feature types:

- Nonlinear features : 5 studies
- Fractal dimension-based features (including multifractal detrended fluctuation analysis, Katz fractal dimension, etc.): 5 studies
- Complexity features (including Lempel-Ziv Complexity, largest Lyapunov exponent, approximate entropy,

fuzzy entropy): 5 studies
- Power features (including power spectral density): 3 studies
- Spectral and bicoherence features : 2 studies
- q-statistics, aperiodic slope/P3a, kurtosis, mutual information : 1 study each
- We didn't find mention of feature type missing for any study

Classification methods:

- Support Vector Machine (SVM, including linear SVM) : 10 studies
- Radial Basis Function Neural Network : 3 studies
- k-Nearest Neighbors : 2 studies
- Probabilistic Neural Network, XGBoost, Convolutional Neural Network (ResNet), CatBoost, ensemble learning, clustering, Decision Tree, Naive Bayes, Boosted Trees, Receiver Operating Characteristic : 1 study each
- Several studies used more than one classification method
- We didn't find mention of classification method missing for any study

Accuracy/performance:

- Reported accuracy or main performance metric was 99% or higher in 3 studies (100%, 99.58%, 99.5%)
- Accuracy was between 94% and 97% in 2 studies
- Accuracy was between 83.33% and 87.5% in 6 studies
- Accuracy was between 77.9% and 78.02% in 2 studies
- Area under the curve was reported as 0.9158 in 2 studies and 0.79 in 1 study
- Sensitivity and specificity (0.71, 0.70) were reported in 1 study
- 1 study reported ”higher than state-of-the-art” without a specific value
- For 2 studies, we didn't find mention of accuracy or performance

Validation approach:

- 4-fold cross-validation: 3 studies
- 10-fold cross-validation: 3 studies
- Area under the curve reported without explicit cross-validation: 2 studies
- Sensitivity/specificity reported without explicit cross-validation: 1 study
- For 7 studies, we didn't find mention of cross-validation or validation approach

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Regional and Spectral Patterns

Regional specificity:

- Frontal and prefrontal regions were most frequently implicated in group differences (e.g., Khoshnoud et al., 2017/2018, Ginard Puigserver, 2016, Gu et al., 2022)
- Some studies reported differences in temporal (Asayesh et al., 2024) or occipital (Valdizán et al., 1997) regions
- Several studies reported global effects (Abramov et al., 2024; Arnett et al., 2021/2022a/2024)

Spectral patterns:

- Where reported, spectral analyses focused on delta, theta, alpha, and beta bands
- Increased theta/beta ratio and altered power in these bands were reported in ADHD (Mao et al., 2025)
- Not all studies reported frequency-specific findings

- The relationship between spectral and fractal/complexity measures was not consistently addressed

Summary of regional and spectral findings:

- Most studies reporting regional effects found the strongest differences in frontal or prefrontal regions
- Spectral findings, when reported, often involved increased theta/beta ratio or altered power in ADHD
- Heterogeneity in methods and reporting limited the ability to draw firm conclusions about the specificity and generalizability of these findings

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Summary

- Distinguishing ADHD from controls:Most included studies reported that fractal and complexity measures derived from multichannel resting-state EEG could distinguish ADHD from neurotypical controls, with particular involvement of frontal and prefrontal regions.
- Classification accuracy:Reported classification accuracy was generally high, with several studies reporting accuracy above 90%. However, these findings are as reported by the studies, and methodological limitations such as small sample sizes and inconsistent reporting were common.
- Direction of effects:Both increased and decreased complexity were reported in ADHD, depending on the metric, region, and paradigm used.
- Methodological limitations:Small sample sizes, inconsistent reporting, and lack of standardization were frequent, limiting the strength of conclusions that can be drawn from these studies.
- Potential as a biomarker:The included studies suggest potential for fractal and complexity measures as biomarkers for neural complexity changes in ADHD, but findings were heterogeneous and should be interpreted with caution given the methodological limitations.

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